Extrema of Functions - Class 1

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Absolute Extrema of Functions

Defintion - Extrema of a Function

A function f has an absolute maximum at c if $f(x) \leq f(c)$ for all x in the domain D of f. The number $f(c)$ is called the maximum value of f on D. Similarly, f has an absolute minimum at c if $f(x) \geq f(c)$ for all x in D. The number $f(c)$ is called the minimum value of f on D. The absolute maximum and absolute minimum values of f on D are called the extreme values, or extrema, of f on D .

- Absolute minimum at $t = c$, $f(c) = \min$ of Range(f).
- Absolute maximum at $t = c$, $f(c) = \max$ of Range(f).

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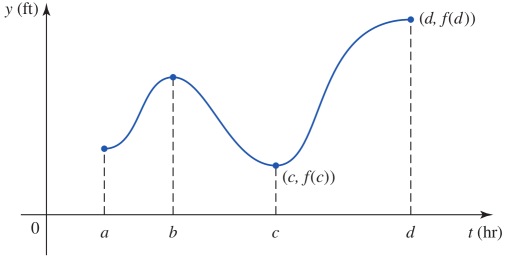
Absolute Extrema of Functions



- The extrema of the function on the domain $[a, d]$
- Absolute minimum at $t = c$, Absolute maximum at $t = d$
- Absolute minimum and/or Absolute maximum can be occur at boudary point

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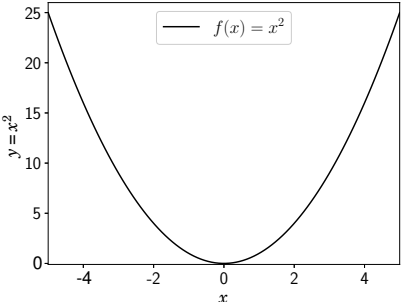
Absolute Extrema of Functions-Example



- $f : \mathbb{D} \to \mathbb{R}$, if $\mathbb{D} = [-3, 3]$, $f(x) = x^2$
- Absolute minimum at $x = 0$ is $f(0) = 0$, Absolute maximum at $x = -3$ and $x = 3$ is $f(-3) = f(3) = 9$

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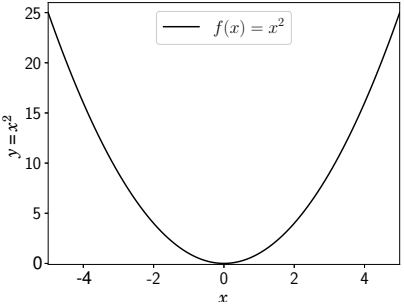
Absolute Extrema of Functions-Example



- $f : \mathbb{D} \to \mathbb{R}$, if $\mathbb{D} = [-3, 1]$, $f(x) = x^2$
- Absolute minimum at $x = 0$ is $f(0) = 0$, Absolute maximum at $x = -3$ is $f(-3) = 9$

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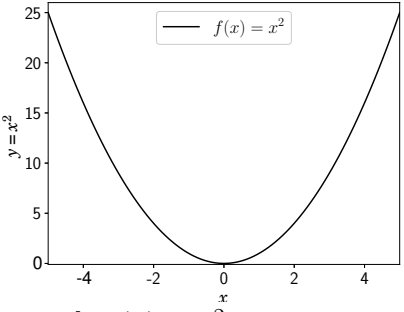
Absolute Extrema of Functions-Example



- $f : \mathbb{D} \to \mathbb{R}$, if $\mathbb{D} = [-4, -1]$, $f(x) = x^2$
- Absolute minimum at $x = -1$ is $f(-1) = 1$, Absolute maximum at $x = -4$ is $f(-4) = 16$

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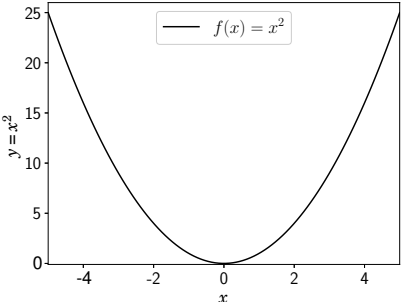
Absolute Extrema of Functions-Example



- $f : \mathbb{D} \to \mathbb{R}$, if $\mathbb{D} = [2, 3]$, $f(x) = x^2$
- Absolute minimum at $x = 2$ is $f(2) = 4$, Absolute maximum at $x = 3$ is $f(3) = 9$

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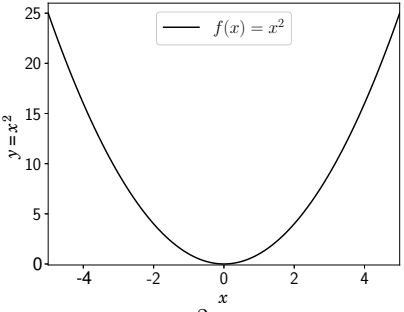
Absolute Extrema of Functions-Example



- $f : \mathbb{D} \to \mathbb{R}$, if $\mathbb{D} = (-1, 2)$, $f(x) = x^2$
- Absolute minimum at $x = 0$ is $f(0) = 0$, There is no Absolute maximum since as x approaches to 2, $f(x)$ increases and approaches 4. But f never attains the value 4 in $\mathbb{D} = (-1, 2)$

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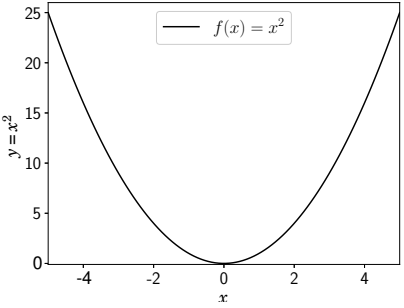
Absolute Extrema of Functions-Example



- $f : \mathbb{D} \to \mathbb{R}$, if $\mathbb{D} = \mathbb{R}$, $f(x) = x^2$
- Absolute minimum at $x = 0$ is $f(0) = 0$, There is no Absolute maximum, since Range$(f) = \mathbb{R}^+ \cup \{0\}$ is not bounded above.

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Absolute Extrema of Functions-Exercises

1. Write an example of a function
   1. where both extrema attained at an interior point.
   2. where both extrema does not exists.
   3. where absolute maxima exists but absolute minima does not exists.
2. $f : \mathbb{R} \setminus \{0\} \to \mathbb{R}$, defined by $f(x) = 1/x$. Find extrema, if exists.
3. sketch the graph of the following functions and find its absolute maximum and absolute minimum values, if any.
   1. $f(x) = 2x + 3$ on $[-1, \infty)$
   2. $g(x) = -3x + 2$ on $(-1, 2]$
   3. $h(t) = t^2 - 1$ on $(-1, 0)$
   4. $f(t) = t^2 - 1$ on $[-1, 0)$
   5. $g(x) = x^2 + 1$ on $(0, \infty)$
   6. $h(x) = x^2 + 1$ on $(-2, 1]$
   7. $f(x) = x^2 - 4x + 3$ on $(-\infty, \infty)$
   8. $g(x) = 2x^2 - 3x + 1$ on $[0, 1)$
   9. $g(x) = \frac{1}{x}$ on $(-1, 1)$
   10. $f(x) = |2x - 1|$ on $(0, 2]$
   11. $h(t) = \cot \pi t$ on $[\frac{1}{4}, 1)$

Voice : Niyati D K

THANK YOU

Software used:
GNU/Linux
TeXLive - Beamer
Kdenlive